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## Third Semester B.E. Degree Examination, June/July 2011 **Engineering Mathematics**

Time: 3 hrs.

Max. Marks:100

Note: Answer FIVE full questions selecting at least TWO questions from each part.

PART - A

a. Find a Fourier series to represent  $f(x) = x - x^2$  from  $x = -\prod$  to  $x = \prod$  and deduce that  $\frac{\Pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ (07 Marks)

b. If 
$$f(x) = \begin{cases} x & 0 < x < \frac{\Pi}{2} \\ \Pi - x & \frac{\Pi}{2} < x < \Pi \end{cases}$$

show that i)  $f(x) = \frac{4}{\Pi} \left[ \sin x - \frac{1}{3^2} \sin 3x + \frac{1}{5^2} \sin 5x - \dots \right]$ 

ii) 
$$f(x) = \frac{\Pi}{4} - \frac{2}{\Pi} \left[ \frac{1}{1^2} \cos 2x + \frac{1}{3^2} \cos 6x + \frac{1}{5^2} \cos 10x + \dots \right]$$
 (07 Marks)

Obtain the Fourier series neglecting the terms higher than first harmonic.

х	0	1	2	3	4	5
y	9	18	24	28	26	20

(06 Marks)

Find the Fourier transform of the function  $f(x) = \begin{cases} x, & |x| \le \infty \\ 0, & |x| > \infty \end{cases}$  where '\alpha' is a positive (06 Marks) constant.

b. Solve the integral equation  $\int_{0}^{\infty} f(\theta) \cos \alpha \, \theta d\theta = \begin{cases} 1 - \alpha & 0 \le \alpha \le 1 \\ 0 & \infty > 0 \end{cases}$ 

Hence evaluate 
$$\int_{0}^{\infty} \frac{\sin^2 t}{t^2} dt$$
 (08 Marks)

- Find the finite Fourier sine transform of f(x) = 2x in  $0 \le x \le 4$ . (06 Marks)
- Form the Partial Differential equation by eliminating the arbitrary function from the 3 equation F  $(xy + z^2, x + y + z) = 0$ Solve:  $xp - yq = y^2 - x^2$ . Solve  $py^3 + qx^2 = 0$  by the method of separation of variable. (06 Marks)

(07 Marks) (07 Marks)

Derive one dimensional heat equation. 4

(07 Marks)

Find the deflections of a vibrating string of unit length fixed ends with initial velocity zero (06 Marks) and initial deflections  $f(x) = k(\sin x - \sin 2x)$ .

c. Solve 
$$\frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2} + \frac{\partial^2 \mathbf{u}}{\partial \mathbf{y}^2} = 0$$
 subject to the conditions

$$u(0, y) = u(1, y) = u(x, 0) = 0$$
 and  $u(x, a) = \sin \frac{n\Pi x}{1}$ . (07 Marks)

## PART-B

- 5 a. Find the real root of the equation  $xe^x = 2$  correct to three decimal places using Newton-Raphson method. (07 Marks)
  - b. Employ Gauss-Siedel iteration method to solve:

$$20x + y - 2z = 17$$

$$2x - 3y + 20z = 25$$

$$3x + 20y - z = 18$$

Carryout 3 iterations.

(07 Marks)

c. Using Power method find the dominant eigen value and the corresponding eigen vector of

the matrix 
$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

(06 Marks)

6 a. Using suitable interpolation formula, find the number of students who obtained marks between 40 and 45. (07 Marks)

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

b. Using divided difference formula to find f(x) given data hence find f(4).

(07 Marks)

-	X	0	2	3	6
	f(x)	-4	2	14	158

c. Using Simpson's  $\frac{1}{3}$ rd Rule to find  $\int_{0}^{0.6} e^{-x^2} dx$  by taking seven ordinates.

(06 Marks)

7 a. State and prove Euler's equation.

(07 Marks)

b. Solve the variation problem  $\sigma \int_0^1 (y^2 + x^2y^1) dx = 0$ , y(0) = 0, y(1) = 1.

(06 Marks)

- c. Find the path in which a particle in the absence of friction will slide from one point to another in the shortest time under the action of gravity. (07 Marks)
- 8 a. Find the z-transform of  $\cosh \theta$  and  $\sinh \theta$ .

(06 Marks)

b. Find the inverse z-transform of  $\frac{z^3 - 20z}{(z-3)^2(z-4)}$ .

(07 Marks)

c. Solve:  $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$  with  $y_0 = y_1 = 0$  using z-transform.

(07 Marks)

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